

Self-completeness and spontaneous dimensional reduction

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Abstract

It has recently been shown via an equivalence of gravitational radius and Compton wavelength in four dimensions that the trans-Planckian regime of gravity may be semi-classical, and that this point is defined by a minimum horizon radius commensurate with M_{Pl} . We generalize this formalism to an arbitrary number of dimensions d , and show that gravity in $d > 3$ dimensions remains self-complete, while in lower dimensions it is not. Most interesting is the case for a $(1 + 1)$ -dimensional dilaton gravity model resulting from dimensional reduction of Einstein gravity, which we show to be self-incomplete with no lower bound on possible black hole masses. Potential phenomenological implications of this result are considered.

Gravity is without any doubt a very non-standard interaction. What is usually easy to understand in the context of the other fundamental forces of nature is otherwise elusive in this arena. The peculiar character of gravity stems not only from the well-known non-renormalizability at short scales but also from another specific feature known as *self-completeness*.

In the standard picture of quantum field theory, shorter length scales of a physical system become visible as one increases the energy of the probe. The Compton wavelength – representing the best possible resolution of the position of a particle – is governed by the well-known relation

$$\lambda_C = \frac{2\pi\hbar}{Mc} , \quad (1)$$

where M is the mass associated to the particle under consideration. The current LHC working energy $\sim 8\text{TeV}$ thus corresponds to matter compressed within the exceedingly miniscule distance of $\sim 10^{-19}\text{ m}$ [1]. In principle one may be tempted to conclude that λ_C can be arbitrarily small, provided enough energy can be supplied to a particle. By probing shorter and shorter distances, however, one enters a regime in which the background spacetime manifold becomes significantly disturbed by the energy involved in the process. Such a disturbance prevents the localization to better accuracies than a fundamental (minimal) length scale ℓ .

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In other words, gravity prevents the compression of matter beyond certain distances due to the formation of a black hole. When

$$\lambda_C \sim r_g \quad , \quad (2)$$

the particle collapses to form a black hole and cannot be made smaller. Here $r_g = 2GM/c^2$ is the gravitational radius associated to the particle mass M . Unlike in the case of the Compton wavelength, a further increase of the energy of the system will result in a bigger gravitational radius. As a result the above condition sets the minimum attainable size when both gravity and quantum mechanics are concerned. Not surprisingly, one finds

$$\ell \equiv r_{g,\min} = 2\sqrt{\pi}\ell_{\text{Pl}}. \quad (3)$$

and correspondingly

$$M_{\text{BH},\min} = \sqrt{\pi}M_{\text{Pl}}. \quad (4)$$

where $\ell_{\text{Pl}} \simeq 1.6 \times 10^{-35}$ m and $M_{\text{Pl}} \simeq 1.22 \times 10^{16}$ TeV/ c^2 are the Planck length and the Planck mass, respectively (see left plot in Fig 1).

The above relations show that gravity is self-complete, namely it protects the ultraviolet regime by setting quantum mechanical limits to length and energy. This result is often employed as an elegant argument to downplay the problem of curvature singularities, which would always be inaccessible due to the presence of an event horizon [2]. This viewpoint has been reinforced by a general result obtained in an independent way by several quantum gravity modifications of black hole metrics [3, 4, 5]: even at the terminal stage of the evaporation the probe of the shortest scales in the vicinity of the curvature singularities would not be possible for the formation of zero temperature black hole remnants, *i.e.*, extremal configurations occurring also in the case of non-rotating, neutral black holes [6].

We now demonstrate the robustness of this line of reasoning by its extension to the case of additional spatial dimensions, a usual requirement when considering the eventual quantum gravity phenomenology at the terascale. In a $(d+1)$ -dimensional spacetime one finds

$$\ell \equiv r_{g,\min} = \left(\frac{4\pi\hbar G_d}{c^3} \right)^{\frac{1}{d-1}} \quad (5)$$

$$M_{\text{BH},\min} = 2^{\frac{d-3}{d-1}} (\pi\hbar)^{\frac{d-2}{d-1}} c^{-\frac{d-4}{d-1}} G_d^{-\frac{1}{d-1}} \quad (6)$$

Here G_d , the d -dimensional gravitational constant, reads

$$G_d = 2K_d \pi^{1-\frac{d}{2}} \Gamma\left(\frac{d}{2}\right) \frac{c^3}{\hbar} \left(\frac{\hbar}{M_d c} \right)^{d-1} \quad (7)$$

where $M_d \sim 1\text{TeV}/c^2$ is the d -dimensional Planck mass and K_d is a constant that varies according to the definitions of M_d . The only requirement for K_d is a matching between G_3 and Newton's constant G , *i.e.*, $K_3 = 1$ for $M_3 = M_{\text{Pl}}$. The other factors in (7) come for the Gauss law in d

dimensions. For the present discussion and without loss of generality we can simply set $K_d = 1$ for all d as in [7]. As a result the limits for energy and length turn out to be

$$\ell \equiv r_{g,\min} = 8^{\frac{1}{d-1}} \pi^{-\frac{1}{2}(\frac{d-4}{d-1})} \left[\Gamma\left(\frac{d}{2}\right) \right]^{\frac{1}{d-1}} \left(\frac{\hbar}{M_d c} \right) \quad (8)$$

and

$$M_{\text{BH},\min} = 2^{\frac{d-4}{d-1}} \pi^{\frac{3}{2}(\frac{d-2}{d-1})} \left[\Gamma\left(\frac{d}{2}\right) \right]^{-\frac{1}{d-1}} M_d. \quad (9)$$

By varying $d = 4 - 10$ we find $\ell = (1.23 - 2.00)\hbar/M_d c$ and $M_{\text{BH},\min} = (\pi - 5.13)M_d$, indicating that microscopic black holes are at the reach of the typical LHC energies (see [8] for the latest production rate estimates).

In the above derivation we did not invoke any specific quantum gravity character other than the fact that the energy involved are of the order of the Planck scale. The issue arises since the spacetime at this length scale might be radically different than its conventional picture. Due to its intrinsic graininess, the quantum spacetime has often been described in terms of a fractal surface which smoothly approaches the structure of differential manifold only in the infrared limit. As a result, the very concept of spacetime dimension becomes ill defined or at the very least needs revising. A reliable indicator of the dimension of a fractal is given by the spectral dimension, *i.e.*, the actual dimension perceived by a random walker. In several approaches to quantum gravity the spectral dimension has a general form like

$$\mathbb{D} = D_{\text{IR}} - \frac{a}{b + (\sigma/\ell^2)} \quad (10)$$

where the infrared dimension is $D_{\text{IR}} = d_{\text{IR}} + 1$, σ is the diffusion time with the dimensions of a length squared, a and b are dimensionless constant depending on the specific model of quantum manifold under consideration [9, 4]. While at large diffusion times, *i.e.*, $\sigma/\ell \gg a, b$, the spectral dimension $\mathbb{D} = D_{\text{IR}}$ as expected, in the opposite regime, *i.e.*, $\sigma/\ell \ll a, b$, the spectral dimension decreases to the ultraviolet value $D_{\text{UV}} = D_{\text{IR}} - a/b$. Such a behavior is connected to the potential power counting renormalizable character of gravity which in two dimensions exhibits a dimensionless coupling constant, as is evident from (7) for $d = 1$. In other words if the quantum spacetime underwent a spontaneous dimensional reduction to $D_{\text{UV}} = 2$ ($d_{\text{UV}} = 1$), the non-renormalizability of gravity would just be an “apparent” low energy, classical effect [10].

Given these constraints, the usual argument for gravity self-completeness has to be reviewed. Eq. (5) and (6) are singular for $d = 1$. We therefore have to reconsider the behavior of r_g in lower dimensions. Before doing this we see that the gravitational potential due to the integration of the Gauss law in $(1 + 1)$ -dimensions reads

$$\phi_1(x) = -2\pi \frac{c^3}{\hbar} Mx + \phi_0, \quad (11)$$

where x is the spatial coordinate and ϕ_0 is an integration constant. The above relation shows that the quantity $G_1 M/c^2$ no longer has the dimension of a length, but rather inverse length. This

implies that the gravitational radius $r_{g,1}$ in $(1+1)$ -dimensions will have a intriguing new behavior, *i.e.*,

$$r_{g,1} \propto \lambda_C. \quad (12)$$

The above result implies that in case of spontaneous dimensional reduction at the Planck scale the conventional arguments in support of gravity self-completeness might be no longer valid and would require further more detailed investigations.

A more rigorous argument for this case may be made by considering a specific (relativistic) model of $(1+1)$ -dimensional gravity able to circumvent the triviality of Einstein equations. This is usually achieved by the so called dilaton gravity (DG) models in which the dilaton, an extra field dated back to Kaluza-Klein theories and reappeared in the context of string theory, accounts for some key features of the higher dimensional theory (for a review see [11]). A generic action for $(1+1)$ -dimensional dilaton gravity is

$$S_{\text{DG}} = \frac{c^4}{8\pi G_1} \int d^2x \sqrt{-g} [\psi R + U(\psi)(\nabla\psi)^2 - 2V(\psi)] \quad (13)$$

where ψ is the dilaton field and the functions $U(\psi)$, $V(\psi)$ are model-dependent potentials (see [12] for a comprehensive tabular summary). For the purposes of the present discussion, however, the choice of theory is not arbitrary. We need to invoke the mechanism of spontaneous dimensional reduction, $d_{\text{IR}} \rightarrow d_{\text{UV}} = 1$, to determine the profile of the above potentials. Starting from the $(d_{\text{IR}} + 1)$ -dimensional action for general relativity

$$S_{(d_{\text{IR}}+1)} = \int d^{(d_{\text{IR}}+1)}x \sqrt{-g} \left(\frac{1}{\kappa_d} R + \mathcal{L}_m^{(d_{\text{IR}}+1)} \right) \quad (14)$$

where $\mathcal{L}_m^{(d_{\text{IR}}+1)}$ is the matter Lagrangian, one can show that, by expanding (14) in powers of $(d_{\text{IR}} - 1)$, the theory reduces to

$$S_{(1+1)} = \int d^2x \sqrt{-g} \left[\left(\frac{c^4}{8\pi G_1} \psi R - \frac{1}{2} (\nabla\psi)^2 \right) + \mathcal{L}_m^{(1+1)} \right], \quad (15)$$

in the limit $d_{\text{IR}} \rightarrow 1$ provided that $\kappa_d = 4\pi(1 - d_{\text{IR}})G_{\text{IR}}/c^4$ [13]. A more general mechanism of dimensional reduction from Einstein gravity to an effective $(1+1)$ -Liouville gravity have been studied, with the addition of extra terms in the action [14]

$$S = \int d^2x \sqrt{-g} \left[\frac{c^4}{8\pi G_1} \psi R - \frac{1}{2} (\nabla\psi)^2 - C e^{-2\psi} + \mathcal{L}_m^{(1+1)} \right] + (\text{boundary terms}) \quad (16)$$

where C is a constant. This enhanced model contains (15) as a subset and provides a robust match for dimensionally-reduced spherical gravity.

The above theories provide a faithful description of gravity in $(1+1)$ -dimensions: they preserve classical and semiclassical properties of higher dimensional gravity [15] and can be simply connected to other dimensional reduced gravity proposals [16]. By solving the corresponding field equations one finds the metric

$$ds_1^2 = - \left(\frac{2G_1 M}{c^2} |x| - C \right) dt^2 + \frac{dx^2}{\left(\frac{2G_1 M}{c^2} |x| - C \right)} \quad (17)$$

where x is the spatial coordinate and the constant C , depending on its sign, correlates to specific physical objects in our results. The horizon is thus

$$r_{g,1} \equiv |x|_H = \frac{c^2}{2G_1 M_{\text{BH}}} C \quad (18)$$

which occurs only for $C > 0$. In this case, the usual self-completeness “condition” results in the mass-independent expression fixing the constant C ,

$$\frac{c^2}{2G_1 M_{\text{BH}}} C = \frac{2\pi\hbar}{M_{\text{BH}} c} \implies C = \frac{4\pi\hbar}{c^3} G_1 \implies C = 8\pi^2. \quad (19)$$

This is a striking result, in that it demonstrates the self-*incompleteness* of gravitation in a two-dimensional spacetime (see right plot in Fig. 1).

The value and sign of C have the following repercussions. First, for $C \neq 8\pi^2$ one has “classical black holes”, *i.e.*, black holes that do not meet the condition for being elementary particles. Second, in a hypothetical trans-Planckian collision ¹, the particle Compton wave length can be made smaller than $\hbar/M_{d_{\text{IR}}} c$, the d_{IR} -dimensional Planck length, resulting in the formation of tiny, non-classical, trans-Planckian black holes (*i.e.* $M_{\text{BH}} > M_{d_{\text{IR}}}$) for $C = 8\pi^2$. Third, when $C < 0$, the horizon is undefined and the object is an elementary particle.

Lastly, we emphasize there is no minimum energy scale that defines a black hole. This is reflected by the fact that for a given mass M_{BH} , it is always possible to have black hole smaller (for $0 < C < 8\pi^2$) or larger (for $C > 8\pi^2$) of the corresponding Compton wavelength of a particle of the same mass. Note this is also true in the case of the extension of the present theory to the case of a $(1+1)$ -dimensional, singularity free, non-commutative geometry: no extremal black hole configurations occur [17]. Unlike in the four-dimensional (or higher) case, sub-Planckian black holes (*i.e.* $M_{\text{BH}} < M_{d_{\text{IR}}}$) are possible in $(1+1)$ -dimensional spacetime. They cannot, however, be the direct product of a dimensional reduction mechanism (which requires Planckian energies), but rather a transient state between the formation of Planckian (or trans-Planckian) $(1+1)$ -black holes and their complete evaporation or some other stable configuration.

We can see this by studying the thermal properties of these dimensionally reduced black holes. From the periodicity of the Euclidean time of (17) one finds the temperature

$$T = \frac{\hbar G_1}{2\pi k_{\text{B}} c} M_{\text{BH}} \quad (20)$$

which becomes $T = M_{\text{BH}} c^2 / k_{\text{B}}$ by using the definition of G_1 . We see that heavier (trans-Planckian) holes are smaller and hotter. However the positive sign of the heat capacity $\mathcal{C} \equiv \partial(Mc^2)/\partial T = k_{\text{B}}$ indicates that the evaporation slows down as the black hole loses energy, the contrary of what

¹In this case, the terms sub- and trans-Planckian follow a colloquial definition as measured by an external observer that supplies the energy for the collision. Since G_1 is a dimensionless quantity, we should not be allowed to define a Planck mass or equivalently a Planck length. This is another way to show that there is no minimum energy scale to define a black hole, as explained below. We have retained the conventional definition, however, since we have distinguished between the spectral dimension – the actual dimension “perceived” by probes (*e.g.* particles) – and the conventional topological dimension, *i.e.* d_{IR} .

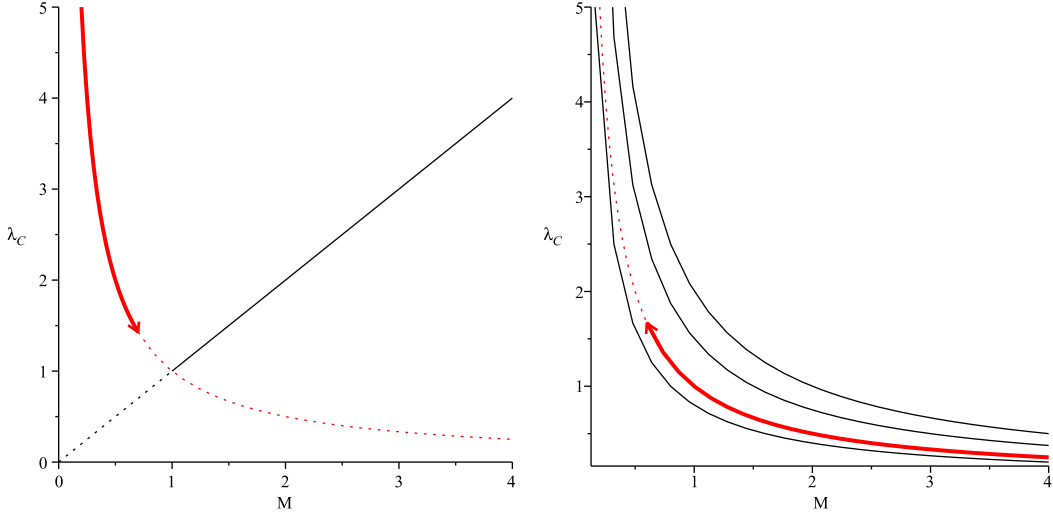


Figure 1: The scale-size in the infrared regime (left) as a function of the particle mass M , including the Compton wavelength λ_C (solid thick hyperbola) and the gravitational radius r_g (solid linear) for classical, trans-Planckian black holes. The intersection of the above curves gives the minimum mass for forming a black hole as well as the minimum detectable length scale as it appears in the infrared limit. The direction of the arrow shows the increase of mass from sub-Planckian to Planckian scales. (Right) Scale-size in the ultraviolet regime as a function of M , where the effective spacetime dimension is 2 and gravitational radii are hyperbolae (thin solid curves). The thick solid curve represents a quantum black hole, *i.e.*, a black hole which is at the same time an elementary particle. Classical as well quantum black holes can occur for any mass. By Hawking emission trans-Planckian black holes decay into sub-Planckian black holes according to the direction of the arrow. In both plots we adopted the units $\ell = 1$, $M_{\text{BH,min}} \sim M_{d_{\text{IR}}} = 1$

happens in the four (and higher) dimensional case. This fact results in a peculiar form of the Stefan-Boltzmann law, which in $(1 + 1)$ -dimension reads [18]

$$-\frac{dM_{\text{BH}}}{dt} \propto T^2 \propto M_{\text{BH}}^2. \quad (21)$$

From the above relation we see that the relative mass loss $\frac{1}{M_{\text{BH}}} \left| \frac{\partial M_{\text{BH}}}{\partial t} \right| \sim M_{\text{BH}}$ increases with the mass. Lighter black holes are bigger, colder and tend to evaporate at a slower rate. We have to restrict our analysis to mass parameters compatible with the case under consideration, however, *i.e.*, initial Planckian (or trans-Planckian) black holes with mass $M_{\text{BH,TP}} \gtrsim M_{d_{\text{IR}}}$ decaying into smaller sub-Planckian holes with mass $M_{\text{BH,SP}} < M_{d_{\text{IR}}}$. As a consequence, we can estimate evaporation times t_{ev} for such a decay by integrating (21) within the corresponding mass interval. As a result we have that

$$t_{\text{ev}} \sim \left(\frac{M_{d_{\text{IR}}}}{M_{\text{BH,SP}}} - \frac{M_{d_{\text{IR}}}}{M_{\text{BH,TP}}} \right) \left(\frac{\hbar}{M_{d_{\text{IR}}} c^2} \right), \quad (22)$$

which, for $M_{\text{BH,SP}} \lesssim M_{d_{\text{IR}}}$, sets the typical time scale associated to the scale $M_{d_{\text{IR}}}$, *i.e.* $t_{\text{IR}} \equiv \left(\frac{\hbar}{M_{d_{\text{IR}}} c^2} \right)$. Smaller holes would have $t_{\text{ev}} > t_{\text{IR}}$, resulting into quasi-stable objects.

To better understand the fate of such sub-Planckian black holes with mass $M_{\text{BH,SP}} \lesssim M_{d_{\text{IR}}}$ we should go back to the mechanisms of dimensional reduction, *i.e.* the phase in which (trans-)

Planckian black holes forms. The process is again regulated by the time scale t_{IR} . The effective spacetime dimension smoothly decreases from D_{IR} to 2 in a time lapse of the order $\sim t_{\text{IR}}$ during which the system undergoes a temporary $(2 + 1)$ -dimensional regime. This corresponds to having Newton's potential of the form

$$\phi_2 = \phi_0 - G_2 M \ln(r/\ell_{\text{Pl.}}) = \phi_0 - 2c^2 \left(\frac{M}{M_{\text{Pl.}}} \right) \ln(r/\ell_{\text{Pl.}}) \quad (23)$$

where we have used the relation $G_2 = 2(c^3/\hbar)(\hbar/M_{\text{Pl.}}c)$ and ϕ_0 is another integration constant (distinct from that in (11)). From (23) we see that the $(2 + 1)$ -dimensional spacetime is a special case in which $G_2 M/c^2$ is a dimensionless quantity, signaling the absence of event horizons. This fact is in agreement with black holes derived from the BTZ action, that can only exist in anti-deSitter universes [19]. In addition Newton's potential is divergent both in the UV and IR regimes, a borderline situation between IR-dimensional and UV-dimensional cases. As a consequence we interpret the $(2 + 1)$ -dimensional case as the regime of a phase transition between the higher-dimensional and the two-dimensional black holes.

In light of this reasoning we can argue that sub-Planckian black holes would not completely evaporate, but they would rather undergo to a “dimensional oxidation”, namely the opposite process to the dimensional reduction. In other words the initial (trans-) Planckian black holes would decay into transient sub-Planckian black holes in a time $t_{\text{ev}} \sim t_{\text{IR}}$ to oxidate into ordinary sub-Planckian particles in the $(d_{\text{IR}} + 1)$ -dimensional spacetime. This might change the conventional signatures for black hole detection in particle accelerators [20] and explain the reason why latest data tend to exclude their observation [21].

In summary, we have presented in this letter new insights on the nature of lower-dimensional gravitation, specifically that the well-known self-completeness condition does not hold in spacetimes below $(3 + 1)$ -dimensions. The immediate consequence of this result is that the mass of a quantum black holes formed in a $(1 + 1)$ -dimensional Universe is unbounded from below. Consequently, this would suggest that evaporating black holes will eventually reach a new non-thermal phase in their evolution if Planckian dimensional reduction theories are correct. For primordial black holes of the appropriate masses, this has already occurred. Such objects can thus be considered a new catalyst in early Universe physics such as formation mechanism, a possible new candidate for dark matter, or even a new bi-product of high-energy collisions at the specified energy. Depending on the scale of such transitions, physical evidence could be just within reach of present or future collider experiments, ultra-high energy cosmic ray detectors, of other cosmological probes.

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